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Identifying Threshold Concepts in Physics: too many to count!

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Abstract

This study used a variety of methodologies to outline the omnipresence of Threshold Concepts in Physics. We found out that interview methods searching for Threshold Concepts have to be discipline-specific. We pointed out that transformative thinking through recursiveness takes place in two directions. Quantitative methods borrowed from Physics Education Research may not work well when used to ascertain the experimental uncertainty as a Threshold Concept.

Keywords: Physics, Experimental Uncertainty, Electromagnetism

Introduction

Pedagogical research in Physics is largely dominated by Physics Education Research (PER). PER places misconceptions at the heart of students' troubles (McDermott and Schaffer 2002, Mazur 1996). Misconceptions commonly relate to conceptually difficult or troublesome knowledge. They are incomplete, contradictory, stable and highly resistant to change. PER methods aim at putting students in a state of cognitive dissonance, thus forcing them to confront and uproot their misconceptions.

The interest in doing a study on Threshold Concepts (TC) in Physics was motivated by the idea of comparing the TC findings with well-known facts from PER.

In Threshold Concepts research, it is a common practice to gather suggestions from instructors (Stokes, King and Libarkin 2007, Zander et al. 2008, Park and Light 2009). Instructors identify students' troublesome areas that meet the criteria for a TC.

Selected concepts are later tested on students through semi-structured (Zander et al. 2008) or structured interviews. (Stokes, King and Libarkin 2007, Land, Meyer and Smith 2008)

We decided not to ask our colleagues, but our students. We interviewed students from different courses (year 1 to year 3). The following studies were carried out:

- Students' interviews:
  - Non-specific ("empirical vague" interview method)
  - Discipline-specific (interview method applied to Electricity and Magnetism)
- A quantitative comparative study of *experimental uncertainty* as a Threshold Concept.

The empirical-vague interview method

Between Sept. 2011 and April 2013, a series of one to one interviews were carried out in two lab courses from the 2^{nd} and 3^{rd} years of study. 26 students were interviewed.

In addition, 8 Teaching Assistants volunteered to answer the same questions.

The Questionnaire is presented in Appendix 1. Questions were not specific to any sub-discipline of Physics. The interviews were carried out with the hope that students will remember on the spot what troubled them mostly and also how they managed to get over the troublesome element.

Some of the answers are included in Table 1 below.

Table 1. Troublesome Concepts in Physics, identified through interviews

Concept (number of answers)	What "clicked"	Comments by students
Latent energy (2)	Repetition in several courses, examples	Being exposed to the material I struggled with on a periodic basis, in different contexts
Special Relativity (1)	Examples	It clicked when I tried to break the problem down in simpler concepts
Electric/magnetic flux (3)	Flux of a fluid	Give me something concrete to imagine the concept in my head
Polar coordinates (2)	Working examples in several math/physics courses	Practice, practice....
Optics (5)	Diagrams, visualization, experiments in the lab	I did all the Optics experiments from the lab. This really helped.
Classical thermodynamics (1)	Statistical Mechanics upper year course	Only after taking the upper year course I understood the concepts from the lower year
Quantum threshold energy (2)	Taking 2 courses in Quantum Mechanics, working on many examples and problems	Use the clearest possible language, make connections to known concepts, explain why each is necessary
Potential energy (3)	Repetition in different contexts, other courses	Graphs, visualization
Waves, wave phase, superposition of waves (4)	Demonstrations, clear examples, math formulation, an animation, a brilliant simulation	In order to "click" on a concept, the student has to be ready ("on the point") when demonstration takes place
Polarization of light (2)	Doing the experiment in the lab	Seeing and playing with polarizers
Electric field, electric field lines (5)	Repetition in different years, clear examples	People do not do demonstrations when teaching 2nd and 3rd year theoretical courses. Do not teach abstract theoretical concepts without examples
Spin and angular momentum (3)	Seeing it in another course	It clicked in an upper year course when I was better at math and I knew more physics
Electric circuits (4)	After taking the Electronics Lab, in 3rd year	I always look back at my notes from the Electronics Lab

Electrons and holes as charge carriers (1)	Doing the Hall Effect experiment in the lab twice: 2nd year and 4th year	I imagined few electrons and many holes as rain falling through air; but many electrons and few holes as air bubbles rising through water.
Data fitting (3)	Studying the linearity of the response of a germanium detector	Understanding this allowed me to piece together the rest of the Error Analysis in my 3rd year!

Discussion on the empirical-vague interview method

Students named a variety of troublesome concepts, mostly related to the other courses they were currently taking. The identification was not appropriate in all cases: some students pointed out entire branches of physics as being troublesome: Special Relativity, Optics, and Classical Thermodynamics. A number of clear characteristics of students' struggle with troublesome areas are presented below:

Transformative thinking was generally acquired by repetition. This was noticed in most answers.

Recursiveness took place in two directions:

- Horizontal, when the concept is seen in several courses of the same level,
- Vertical, when the concept is seen in upper level courses.
-

Intuition was built on visual elements or imagined scenarios.

Several students pointed out the need of being 'ready' when the clicking takes place through a demonstration or experiment. This 'state of readiness' seems similar to the *post liminal states* leading to transformation, irreversibility and crossing boundaries defined by the TC literature (Land, Meyer and Smith 2008, Cousin 2006).

It was somewhat expected by the interviewer to obtain information on several related concepts within the same sub-discipline. This is *the integrative aspect of a threshold concept*. Examples from the table above are:

- *Electric and magnetic flux*. If the flux concept is not acquired properly, neither flux of the electric/magnetic fields, nor the flux of particles could be understood. One student pointed out the visualization of flux in the case of a fluid. He successfully applied this general visualization to understanding the flux of the invisible electric or magnetic fields.
- *Waves, wave phase, superposition of waves*. These concepts work together, in general, but if the fundamental one (wave) is not fully understood, all the other do not make sense. All students mentioned a beautiful computer simulation that clicked for them in the classroom.
- *Electric field, electric field lines* concepts also work together but none precedes the other.

It was also expected that much of the troublesome concepts understanding should take place in a lab environment. This was indeed mentioned in many of the answers.

Answers to Question 2: “What was the element from your learning that helped you ‘click’ on concept A (or B)?” could be summarized as:

- Repetition in several courses, same year of study,
- Repetition in higher level (upper year) courses,
- Seeing examples, demonstrations and simulations,
- Doing experiments.

The Teaching Assistants (TA) provided a different array of answers, from undergraduate to graduate concepts. Their answers to Question 2 were more elaborate, but centred on the same four elements mentioned above.

One TA noted:

> *I feel that in our teaching role, we have a tendency to forget the difficulties we had in understanding troublesome concepts. I found that re-visiting those concepts and putting myself in the position of the student can be difficult, but normally yields better results when trying to explain various difficult concepts*

The discipline-specific interview method

In March 2014, 42 students from a second year theoretical physics course: PHY250 Electricity and Magnetism were interviewed using the same questionnaire as before (see Appendix 1). In addition to the questions, each student was given a table with troublesome concepts identified by the instructor (see Appendix 2).

Overwhelmingly, students have selected three groups of troublesome concepts from Electromagnetism. In the decreasing order of number of votes they were:

- Potentials (electric and magnetic)
- Boundary conditions (for the electric or magnetic fields)
- Fields (electric and magnetic)

In explaining their difficulty in grasping *the magnetic potential*, students pointed out that the concept is abstract, does not present any intuitive breakthrough, is loaded with a heavy mathematical apparatus, and was never heard of before.

The *electric potential* was known to students from the introductory physics courses which generally teach concepts based on intuitive approaches. The level of abstraction introduced in the second year course was largely based on the mathematical formulation. Only three students mentioned that the electric potential “clicked” for them, because of the repetitive practice through problems.

Boundary conditions were taught twice in PHY250: once for the electric field, the second time four weeks later for the magnetic field. Students found them conceptually confusing, “not flowing with the course”, with very few practice problems in the textbook. The repetition of the method did not make any difference for the majority of the students who answered the questionnaire. Only two students mentioned that seeing the method again helped them to finally grasp the concept.

Fields (electric and magnetic) were seen as troublesome concepts before (see Table 1). Some students claimed that the electric field was not clear in the first year course. When they met it again in PHY250, the presence of math made things worse.

The discipline-specific interview method did not provide a large array of answers. The methodology was tight and rigid: the table with troublesome concepts was given to students and they were asked to provide an identification of trouble area right away. This led to less spontaneity and the fact that the instructor and interviewer was the same person led to less sincerity.

A quantitative case study: the experimental uncertainty as a Threshold Concept

Overview

We pay a special attention to teaching data analysis in our lab courses. Part of data analysis is the study of uncertainties in experimental physics, based on the fact that there is almost no measurement from simple to very complex without an uncertainty. The study of uncertainties is also named Error Analysis.

Error Analysis is based on the concept of “measurement uncertainty”. International definition of measurement uncertainty is provided by the International Organization for Standardization (ISO 1993): “Parameter associated with the result of a measurement that characterizes the dispersion of the values that could be reasonably attributed to the measurand”.

Understanding of measurement and uncertainty by physics students was extensively studied by several Physics Education Research (PER) groups. Allie et al. 1998, Volkwyn et al.2008, Buffler at al. 2008 investigated the understanding of the nature of scientific measurement by students from non-major or major physics programs at the University of Cape Town, South Africa.

It has been found by Abbot (2003) and also by Lippmann (2003) that undergraduates from two American universities (NCSU and Maryland) have some ability to manipulate errors in math functions, or calculate mean value, standard deviation and standard error of the mean. However, many of them failed at interpreting the distribution of values in repeated experiments.

Progression of students' ideas concerning experimental data was studied by comparing pre- and post-answers to a set of six questionnaire items (Allie et al. 1998, Volkwyn et al. 2008, Buffler at al. 2008). Two probes each on: reasons for repeating measurements, data processing and data comparison were analysed by Volkwyn et al. (2008) and by Buffler et al. (2008). Compatibility to the point paradigm (data are treated in a local realistic manner) or set paradigm (each data provides incomplete information about the measurand) was the criterion used in classification of responses.

The point paradigm reasoning assumes that:

- the datum is the true value,
- any deviation from the result is due to mistakes by the experimenter or environmental factors.

The set paradigm reasoning is in agreement with the nature of scientific measurement:

- each reading is an approximation of the measurand,
- all readings are used together to build the best approximation of the measurand,
- an interval of uncertainty is constructed together with the best approximation of the measurand.

In their analysis, Allie et al. (1998), Wolkwyn et al. (2008) and Buffler et al. (2008) assessed the data in four categories:

- consistent point reasoning,
- mixed reasoning,
- consistent set reasoning,
- unclassifiable reasoning.

Answers by the "mixed reasoners" revealed that they used certain algorithms without making sense of the correct data analysis sequence, represented by the set paradigm statements.

Wilson et al. (2010) have identified the measurement uncertainty as a Threshold Concept in Physics. The identification process took place in a one-day brainstorm meeting with five physicists from four Australian universities. The process assessed all the characteristics of a threshold concept: transformative, integrative, irreversible, boundary-making and troublesome. They found that the measurement uncertainty meets all of them.

It is a common fact in the threshold concepts literature that instructors tried to use their own experience to assess students' difficulties in grasping troublesome concepts. Some of these concepts were being carefully identified to belong to the threshold type (Davies 2009). According to Wilson et al (2010), there are 5 stages of understanding of uncertainty, presented in Table 2:

Table 2. Stages of understanding of the experimental uncertainty

Stage 1	No conception of uncertainty, no thought of it in relation to experimental outcomes *"I did an experiment and got this answer which is correct!"*
Stage 2	Uncertainty is seen as mistakes *"I did an experiment twice and got a different answer every time so I probably made a mistake or my instruments are broken"*
Stage 3	Uncertainty is seen as a mean of quantifying how wrong you are *"I know the right answer from the book, so my measurement is wrong"*
Stage 4	Uncertainty is seen as something that must be planned for *"I have to take many measurements in order to assess the uncertainty"*
Stage 5	Uncertainty is a comprehensible, quantifiable result *"I have to calculate the mean value and quantify the spread of variables"*

Wilson et al (2010) carried out semi-structured interviews with 24 randomly selected first year students from four universities. Students were asked to compare data sets, assess data spreads and identify factors that contributed to data scatter. Wilson et al's study suggested that very few students were able to quantify coherent ideas about data spread, but no quantitative data were provided to support this conclusion.

A quantitative study of experimental uncertainty

At the Department of Physics, University of Toronto, we introduce the experimental uncertainty in first year laboratories and 'Practicals' settings. We teach: distribution of values in repeated experiments, types of errors, mathematical manipulations, etc., several times in the first and second year.

In the second year of study, we teach the theory of uncertainties again, in a lab course environment. We introduce new elements and we use computation to implement the advanced concepts (Serbanescu, Kushner and Stanley 2011). Students do a number of specially designed exercises aimed at correlating the theory of Error Analysis with practical experimental situations.

In order to assess students' knowledge, the PER methodology of using pre- and post-instruction tests was used. Two Error Analysis quizzes were given at a six weeks interval. The tests included five questions: the first two were conceptual and carried 1 grade each. The others were numerical problems with 4 grades each. The tests were each worth 10% of the final grade of the course.

Assessing experimental uncertainty as a Threshold Concept (TC)

The following TC Question was identical in both tests. It was written by following the stages of understanding of uncertainty found by Wilson et al (2010) and presented in Table 2. Stages 2 to 5 correspond to options a) to d), below:

How would you define the experimental uncertainty? Choose the statement that applies best in your opinion:

a) *Uncertainty quantifies the mistakes you do*
b) *Uncertainty quantifies how wrong you are*
c) *If you make sufficient repeated experiments you can determine the uncertainty*
d) *Parameter attributed to a measurement which quantifies the variability in the method.*

Students' answers to the TC Question were correlated to the test grades. Records missing one of the two tests were deleted. The final sample size was 70. Figures 1 and 2 show the results of the analysis in boxplot format. The third quartiles are displayed with a lighter shade than the second quartiles. The line separating the second from the third quartile is the median. The upper (lower) error bar is the distance between the largest (lowest) value and the third (second) quartile box.

Figure 1. Boxplots of Test 1 (pre-test) grades over answers to the TC Question. Answers a) - d) mean: a) = least knowledge to d) = most knowledge.

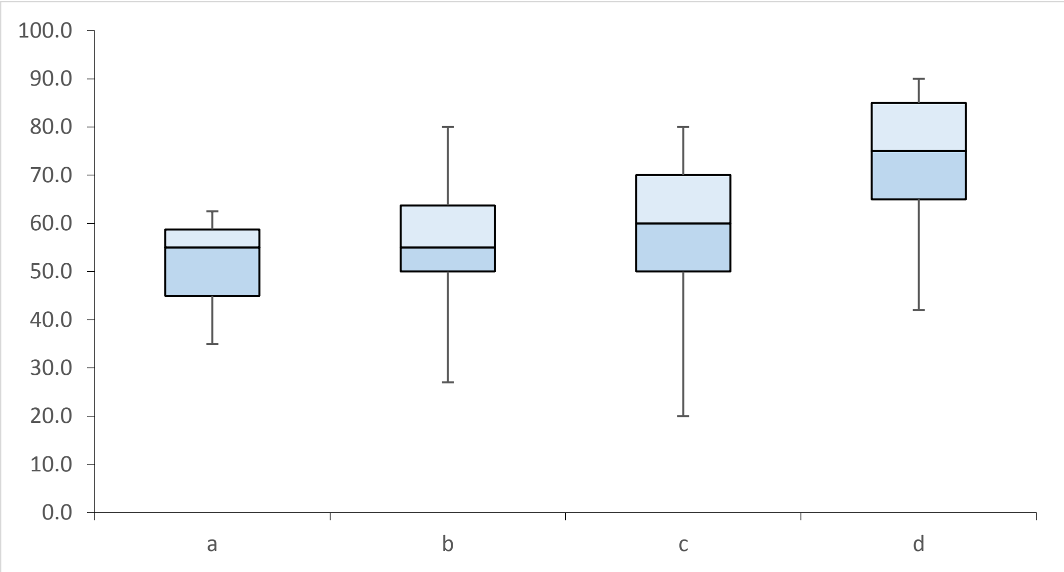
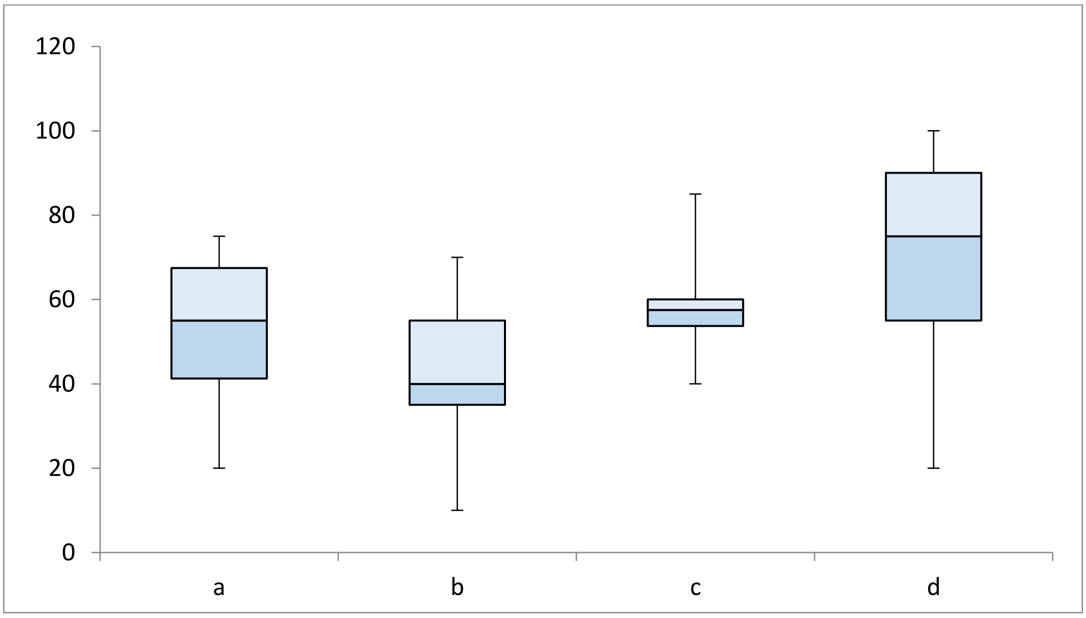


Figure 2. Boxplots of Test 2 (post-test) grades over answers to the TC Question. Answers a) – d) mean: a= least knowledge to d = most knowledge.



Analysis and comments on the experimental uncertainty studied as a TC

A comparison between the answers to the TC Question in the pre- and post-tests reveals that the number of students who answered a) or b) stayed constant (17) regardless the enhanced instruction. On the other hand, the number of students who provided the right answer d) increased from 36 to 45, but at the same time the number of students who choose the wrong answer a) increased from 3 to 10.
30 students (42.8% of the class size) provided the right answer to the TC question in both tests. This group scored better than the class average in each of the two tests. Data presented in Figures 1 and 2 apparently show that the intensive instruction that took place between Test 1 and Test 2 did not have a significant effect on students' understanding of the concept of uncertainty.

Confirming Experimental Uncertainty as a PER finding

Another conceptual question in our error analysis tests was meant to assess students' understanding of agreement of data and comparison of data sets and also students' capability to effectively use the concept of experimental uncertainty in a practical task. The PER question was written according to Allie et al. (1998) and Abbott (2003):

Two groups of students measure five releases of a ball in a free-fall experiment. Their results are:

Group A: 118 125 120 128 124, Average = 123
Group B: 121 127 122 124 131, Average = 125

Choose one of the following:

a) *The results from groups A and B are different, since the averages are different.*
b) *The results from group A and B are different because the averages are different and the spreads of data are different.*
c) *The results from groups A and B are not different because the spreads of data overlap to a great extent.*

d) The results from group A and B are not different because the average of A falls well in the range of B and average of B falls well in the range of A."

Figures 3 and 4 show the analysis of answers to the PER Question from Test 1 (pre-test) and Test 2 (post-test). Students' answers to the PER Question were correlated to the test grades. Records missing one of the two tests were deleted. The final sample size was 70. Figures 1 and 2 show the results of the analysis in boxplot format. The third quartiles are displayed with a lighter shade than the second quartiles. The line separating the second from the third quartile is the median. The upper (lower) error bar is the distance between the largest (lowest) value and the third (second) quartile box. The answers a)-d) mean: a = least knowledge to d = most knowledge.

Figure 3. Boxplots of Test 1 grades vs. answers to the PER Question

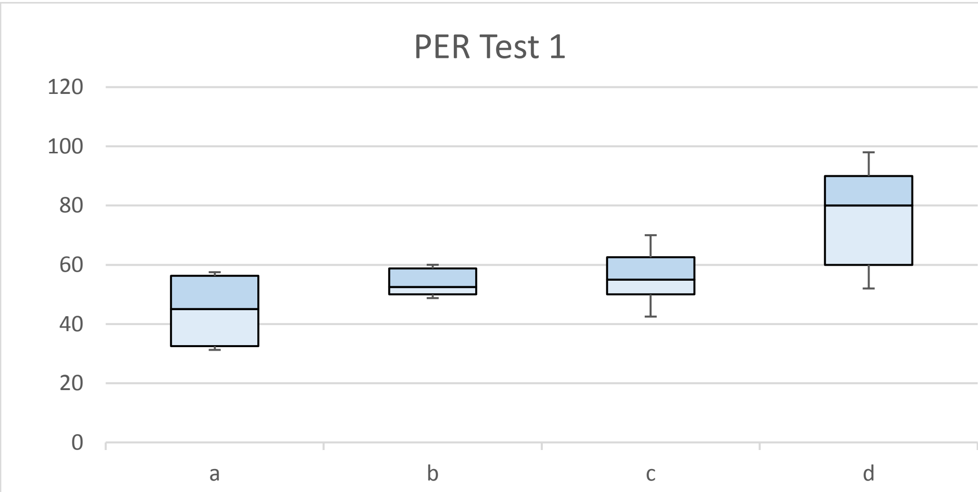
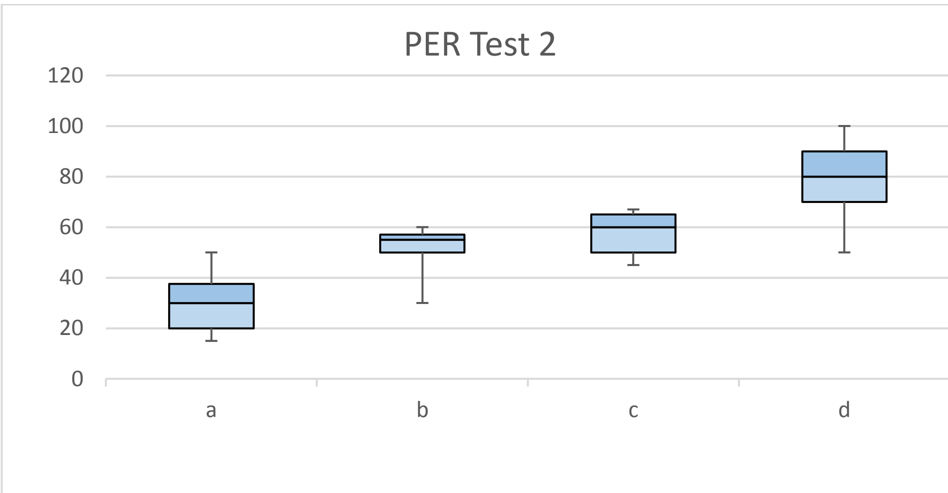


Figure 4. Boxplots of Test 2 grades vs. answers to the PER question



Analysis and comments on the Experimental Uncertainty as a PER concept

We looked at the answers to the PER Question in Tests 1 and 2. We looked at correlations inside each test. PER separates students ('reasoners') into: point paradigm, mixed, set paradigm and unclassifiable.

Our results would place students into the following groups:

- 'a' answerers into the point paradigm group,
- 'b' + 'c' answerers into the mixed group
- 'd' answerers into the set paradigm group.

Between Test 1 and Test 2 the point paradigm group decreased from 18 to 11, but the mixed group increased from 9 to 20.

This suggests that our intensive teaching resulting into moving a number of students from the 'no knowledge' territory to the muddled in-between. These students still do not fully understand uncertainty but can easily manipulate the numerical values into complicated error propagation formulae.

34 students (48% of the class size) provided the right answer to the PER Question in both tests. The average Test 2 grade of this group was: (Mean $\pm$ $\sqrt{N}$) 79.7 $\pm$ 5.9, higher than the entire class Test 2 average (64.4 $\pm$ 8.3).

A T-test was performed to check the correlation of Test 1 and Test 2 grades. Calculated $p = 0.34$ showed a poor correlation.

Comparative discussion on experimental uncertainty as a Threshold Concept or as a Physics Education Research concept

Wilson et al's theory (Wilson et al. 2010) and the PER findings (Allie et al. 1998, Volkwyn et al. 2008, Buffler et al. 2008, Abbott 2003) are very similar. Wilson et al's stages of understanding of uncertainty overlap with the two stages of point paradigm plus three stages of set paradigm from Allie, et al. (Table 3):

Table 3. Comparison of experimental uncertainty formulation in Threshold Concepts and Physics Education Research.

TC (Wilson et al, 2010)	PER (Allie 1998, Abbott 2003, Volkwyn 2008)
Stage 1: No thought of uncertainty in relation to experimental outcomes	**Point paradigm reasoning:** - The datum is the true value - A deviation from the result is due to mistakes by the experimenter or environmental factors, including instruments
Stage 2: Uncertainty is seen as mistakes	
Stage 3: Uncertainty is seen as a means of quantifying how wrong you are	
Stage 4: Uncertainty is understood as something that comes from both user and instrument	**Set paradigm reasoning:** -Each reading is an approximation of the measurand -All readings are used together to build the best approximation of the measurand -Un interval of uncertainty is constructed together with the best approximation of the measurand.
Stage 5: Uncertainty is a comprehensible result that quantifies the variability that can be found in the measurement of the value	

Wilson et al's theory (Wilson et al. 2010) cannot be proved clearly through this study. Transformative thinking cannot be assessed through multiple choice tests, based on identifying key elements from a dry set of definitions.

Much more has to be done to fully confirm that experimental uncertainty is a TC: interviews with students who performed poorly, knowledge assessment through practical tasks, rephrasing the test questions. Fresh TC assessment data (Fall 2016) collected by using a different methodology, provided a new insight into students' reasoning as they were answering the TC Question. It was confirmed that the question itself rather revealed the constant capability of better students to carry a coherent discourse. A positive outcome of this study is that 41% – 42% of class is doing well. This is much better than what others reported (Wilson et al. 2010, Volkwyn et al. 2008).

PER theory worked quite well. Groups of answers from Figures 3 and 4 correspond well to point paradigm, mixed reasoning and set paradigm. This kind of testing can be done in class by directly monitoring students' reasoning and by interviewing them in real time. The traditional physics curriculum relegates uncertainties to labs alone. Improving lab experiences with a few lectures, examples and tutorials is usually not enough. These activities do not engage students' deeper learning. The threshold hurdles cannot be overcome.

For many of our students, the process of identifying, quantifying and propagating uncertainties is a tedious, mystifying chore, perceived as a distraction from the real business of getting an experimental result. This is largely due to the lack of understanding of experimental uncertainty. Explicit pedagogical interventions are required in our case to provide students with the tools and help to acquire this troublesome concept.

Conclusion

Our study pointed out that transformative thinking needed to pass a threshold in Physics is acquired in many cases by repetition (recursiveness). We identified a horizontal direction in recursiveness (courses taken in the same year of study) as well as a vertical direction (courses from different years). The study showed that standard methods used in Physics Education Research may not be successful in identifying a Threshold Concept in Physics. A different, softer methodology is needed, combining brief interviews during an activity (tutorial, lab experiment) with quantitative questions given at the beginning and end of the activity.

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Appendix 1 Questionnaire used in students' and teaching assistants' interviews

Interviews with students and TAs

We believe that a number of Physics concepts are troublesome for students (hard to understand). Some examples are listed below:

- Potential energy
- Gravity
- Electromotive force

1) From your experience, can you name one or two concepts (A, B) you had difficulty understanding in the first place, but then became clear?

2) What was the element from your learning that helped you 'click' on concept A (or B)?

3) Would you be able *now* to teach/explain concept A(B) to another student having difficulty understanding it?

4) Did your full understanding of A(B) helped you understand other concepts later in the course?

5) After fully understanding A(B) did you revisit the learning element that helped you, in order to gain understanding of other concepts?

6) Based on your experience, what advice would you give us to help other students overcome the threshold of A(B)?

Appendix 2 Table given to students interviewed using the empiric discipline-specific method (2014)

PHY250H1S Electricity and Magnetism 2014

Here are some difficult examples from Electricity and Magnetism. Please check the most troublesome concepts(s), both from the point of view of the mathematical tools involved and the physics involved. Please explain in detail how you managed to fully understand some of these concepts.

Concept	Physics	Math
The Dirac delta function		
The electric field		
Electric field of a continuous distribution		
Divergence of E and Gauss's Law		
Curl of E and definition of electric potential		
Boundary conditions in electrostatics		
Electric field inside a conductor		
Force on a conductor		
Magnetic force on a current		
Magnetic field of a current		
Magnetic vector potential		
Boundary conditions in magnetostatics		
Magnetic moment		
Motional emf		
Electromagnetic induction		
Faraday-induced electric field		